Lecture 1 – The Slope of a Straight Line

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1 The Slope of a Straight Line

Motivation: Rate of Change

How fast is the value of a function changing at a given point?

This rate of change is familiar to us for straight lines and is known as the **slope of a line**.

Slope-Intercept Form

Recall the **slope-intercept form of a line** for non-vertical lines:

$$\boxed{y = mx + b}$$

where m is the slope of the line and $(0, b)$ is the y-intercept.

In this class, we focus on the slope of a straight line and the point-slope form of a line.

The slope measures the **steepness** of the line.

Comparing Two Slopes

To see why slope is also called a **rate of change**, compare

$$y = 3x + 7 \qquad \text{and} \qquad y = 6x + 7.$$

For $x = 1, 2, 3, 4$, we obtain

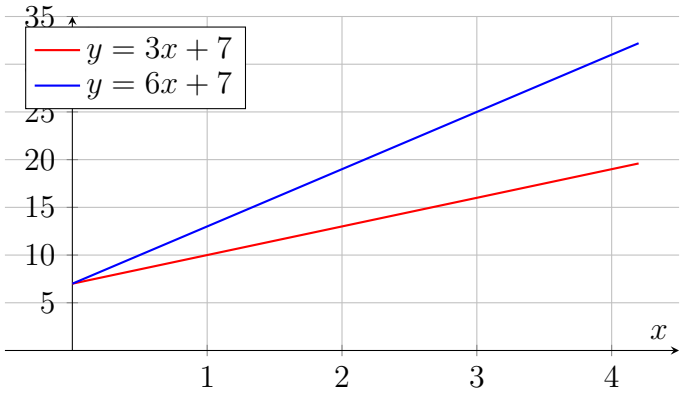
x	1	2	3	4
$3x + 7$	10	13	16	19
$6x + 7$	13	19	25	31

For the first line, whenever x increases by 1, y increases by 3.

For the second line, whenever x increases by 1, y increases by 6.

Different Slopes Give Different Rates of Change

Different slopes give us different rates of change.



Positive and Negative Slope

Consider also the line

$$y = -3x + 7.$$

Here the slope is $m = -3$, which is negative.

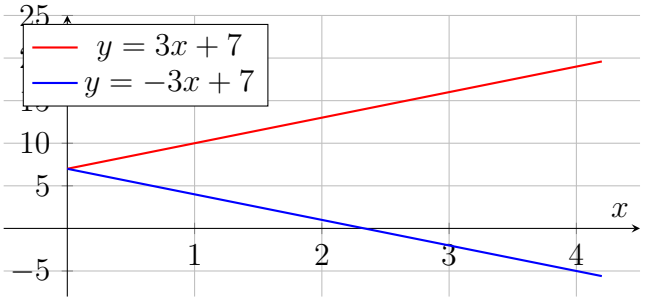
Whenever x increases by 1, the value of y decreases by 3.

x	1	2	3	4
$y = -3x + 7$	4	1	-2	-5

Positive and Negative Slope

$$\boxed{m > 0 \implies \text{the line goes up from left to right}}$$

$$\boxed{m < 0 \implies \text{the line goes down from left to right}}$$



1.1 The Slope of a Line

Given two distinct points (x_1, y_1) and (x_2, y_2) on a line, the **slope of the line** is given by

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

provided $x_1 \neq x_2$.

Think of this as

$$\boxed{m = \frac{\text{change in } y}{\text{change in } x}.}$$

1.2 Point-Slope Form

If we know two points of a line, using the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

we can find the slope of the line. However, we can find something more: **the whole equation of the line.** This is called the **point-slope form** of a line.

Slope-Point Equation of a Line

If we are given two points, we can first calculate

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

and then use the **slope-point equation**

$$\boxed{y - y_1 = m(x - x_1)}$$

to find the equation of the line.

It is not a new type of line. It is simply another way to write the equation of a line when we know a point on the line and its slope.

2 Sketching Lines

3 Example 1 – How to Sketch a Line

To sketch the graph of a line, it is enough to find two points on the line.

1. Choose two convenient values of x, say $x = x_1$ and $x = x_2$.
2. Substitute these values into the equation of the line and calculate
$$y_1 = f(x_1), \qquad y_2 = f(x_2).$$
3. Plot the two points (x_1, y_1) and (x_2, y_2) on the coordinate plane.
4. Draw the straight line passing through these two points.

Example 1 (a)

Sketch the graph of the line

$$y = 3x + 1.$$

Find two points

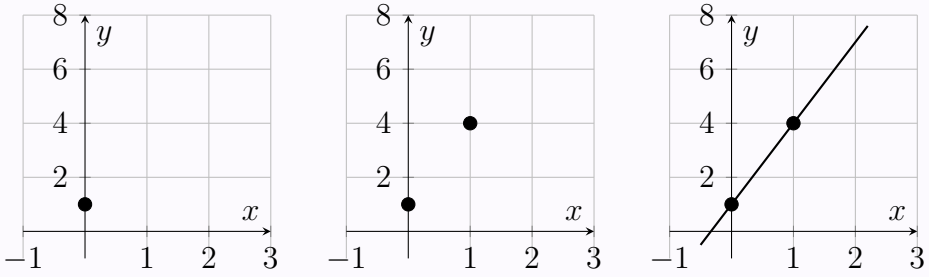
We choose two convenient values of x.

- If $x = 0$, then $y = 3(0) + 1 = 1$. Thus, one point is $(0, 1)$.
- If $x = 1$, then $y = 3(1) + 1 = 4$. Thus, another point is $(1, 4)$.

Therefore, we first plot one point, then both points, and finally draw the line passing through them.

Plot the line

(1) First point (2) Two points (3) Draw the line



The graph is the line through $(0, 1)$ and $(1, 4)$.

Example 1 (b)

Sketch the graph of the line passing through $(2, 3)$ with slope

$$m = -\frac{1}{2}.$$

Find the equation of the line

First, we use the point-slope form:

$$y - y_1 = m(x - x_1).$$

Substituting $(x_1, y_1) = (2, 3)$ and $m = -\frac{1}{2}$, we obtain

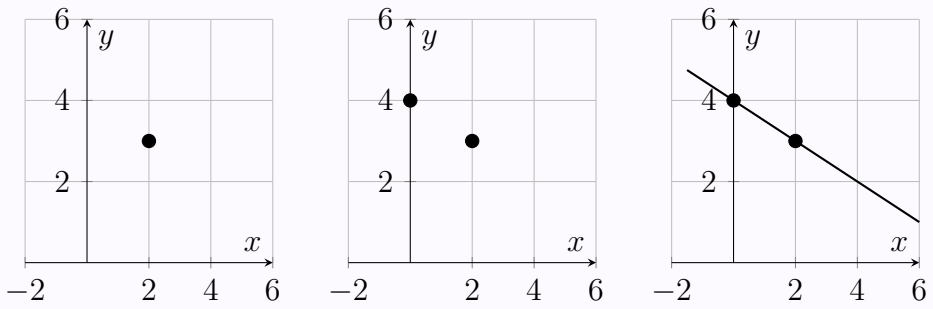
$$\boxed{y - 3 = -\frac{1}{2}(x - 2)}$$

which is the slope-point equation. Now **chooce two points**.

- If $x = 0$, then
$$y - 3 = -\frac{1}{2}(0 - 2) = 1,$$
so $y = 4$. Therefore, $(0, 4)$ is a point on the graph.
- If $x = 2$, then
$$y - 3 = -\frac{1}{2}(2 - 2) = 0,$$
so $y = 3$. Thus, another point of the graph is $(2, 3)$.

Sketch the line

(1) Given point (2) Two points (3) Draw the line



The graph is the line $y = -\frac{1}{2}x + 4$.

Question

Quick Question

What feature of these two lines indicates that the first line should go upwards and the second should go downwards?

Answer

The sign of the slope.

$$m > 0 \Rightarrow \text{line goes up}, \qquad m < 0 \Rightarrow \text{line goes down}.$$

4 Example 2 – Finding the Equation of a Line

Example 2

Find the equation of the line passing through the point $(1, 4)$ with slope

$$m = \frac{2}{3}.$$

Find the equation

We use the point-slope formula

$$y - y_1 = m(x - x_1).$$

Substituting $(x_1, y_1) = (1, 4)$ and $m = \frac{2}{3}$, we obtain

$$\boxed{y - 4 = \frac{2}{3}(x - 1)}$$

This form is **sufficient**. You don't have to simplify if it is not needed.

Example 2 (Completely Optional) Simplifying and Sketch

We can simplify the last equation even more:

$$y - 4 = \frac{2}{3}x - \frac{2}{3}.$$

Therefore,

$$y = \frac{2}{3}x + \frac{10}{3}.$$

So

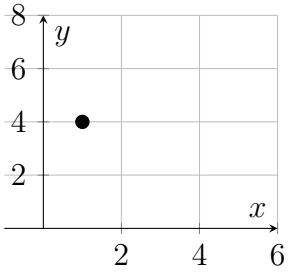
$$\boxed{y = \frac{2}{3}x + \frac{10}{3}.}$$

To sketch the line, we already know the point $(1, 4)$. Choose, for example, $x = 4$. Then

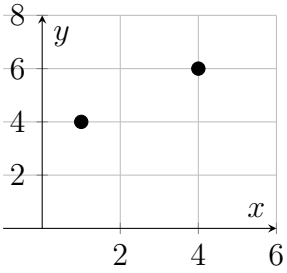
$$y = \frac{2}{3}(4) + \frac{10}{3} = \frac{18}{3} = 6.$$

Thus, another point on the line is $(4, 6)$.

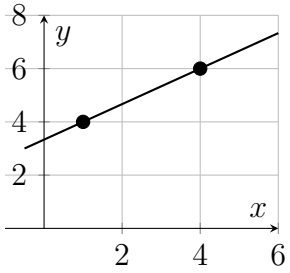
(1) Given point



(2) Two points



(3) Draw the line



5 Example 3 – Equation of a line through two points

Example 3

Find the equation of the line passing through the points

$$(1, -2) \qquad \text{and} \qquad (2, -3).$$

Find the slope

To find the equation of the line, we need the **slope and a point of the line**. Since we know two points, we can apply

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

to find the slope.

Find the slope: Using

$$(x_1, y_1) = (1, -2), \qquad (x_2, y_2) = (2, -3),$$

we obtain

$$m = \frac{-3 - (-2)}{2 - 1} = \frac{-1}{1} = -1.$$

Now that we know the slope, we use the point-slope form

$$y - y_1 = m(x - x_1).$$

Find the equation

Using the point $(1, -2)$, we get

$$y - (-2) = -1(x - 1).$$

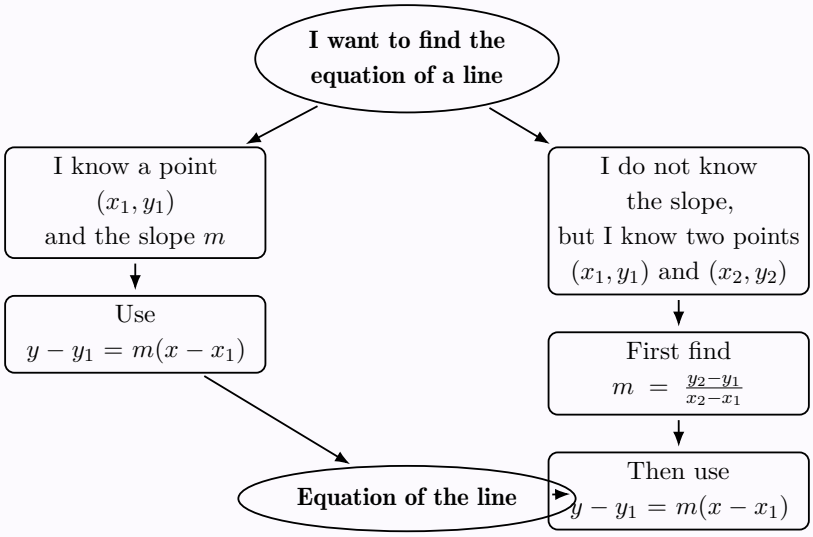
Therefore,

$$y + 2 = -x + 1,$$

and hence

$$\boxed{y = -x - 1.}$$

How do we find the equation of a line ?



6 Parallel and Perpendicular Lines

Parallel and Perpendicular Lines

Let

$$L_1 : y = m_1 x + b_1 \qquad \text{and} \qquad L_2 : y = m_2 x + b_2$$

be two lines.

- L_1 and L_2 are **parallel** if they have the same slope:

$$\boxed{m_1 = m_2}.$$

- L_1 and L_2 are **perpendicular** if the product of their slopes is -1:

$$\boxed{m_1 m_2 = -1}.$$

7 Homework Practice – Lecture 1

Exercise 1 (Homework 1, Problem 1)**.** Find an equation of the given line.

$$\text{Slope is } -3; \quad (1,7) \text{ on line.}$$

What is an equation for a line that has a slope of -3 and passes through the point $(1,7)$?

Exercise 2 (Homework 1, Problem 2)**.** Find the equation of the given line.

$$\text{Slope is } \frac{7}{8}; \quad (3,1) \text{ on line.}$$

Type your answer in point-slope form, using the given point.

Exercise 3 (Homework 1, Problem 3)**.** Find an equation of the given line.

$$\left(\frac{4}{5}, 4\right) \text{ and } \left(-\frac{4}{5}, 3\right) \text{ on line.}$$